УДК 621.01(06)

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THE NUMERICAL METHOD OF WORM GEAR SURFACE DEFINING

To check the accuracy of a worm or wormwheel and to carry out analysis, it is necessary to describe their respective surfaces. The tool action surface should be the same as the surface of the worm which is to mate with the wormwheel workpiece. This condition is not always met, and the worm gear tooth surface is often described approximately. The paper presents a numerical method for generating the surface of a worm gear to be machined in multiple passes with a ball end finger-type cutter by the Lace Cut method. This method enables the formation of worm gears of any arbitrary profile. The advances in the technology of machining worm gears of an arbitrary profile will contribute to the development of new designs of concave-convex worm gears. In the case of small series production or the need for making an additional gear, this method will be the best.

Keywords: *worm gear, wormwheel, ball end mill, Lace Cut method.*

Introduction

Worm gears are used in many machines and devices. Worms are machined by the hobbing method using rotary tool of a disc, finger, ring and cup types. The tool shapes the worm convolution flank over the whole profile height [1, 2]. A wormwheel is machined with a hob by either the tangential or radial method [3]. The hob action surface should be identical to the surface of the worm (taking into account the backlash in the gears) which will mate in the worm gear with the wormwheel being machined. This condition is not always met because of the fact the geometry and technology of hobs are very complex. This is even the case for the cone-derivative worm gear, where the worm is machined using a tool with a rectilinear axial profile of the action surface. Therefore, wormwheels are often machined by the tangential method with a single cutter, or by the radial method with a modular hob (surfaces of the hob and worm are generally different). The worm convolution surface can be described by general and universal equations [4]. The wormwheel tooth surface is often described in an approximate manner. This has an important effect on the modelling accuracy and the examination of worm and wormwheel mating. This is a particularly important problem in the case of more geometrically complex worm gears, such as, e.g., torus-derivative worm gears in which the gear is machined using a tool with a circular axial profile of the action surface. All the more that concave-convex (C-C) worm gears have better operation properties compared to cone-derivative worm gears, and are more and more often used [5]. Defining the surface (profile) of the worm coils and wormwheel teeth is essential not only for the analysis of their meshing, but also to determine the accuracy of their execution in their manufacture process.

Moreover, the application of the method of surface machining in multiple passes with a ball-end finger-type mill, as is the case for machining geometrically complex forms and surfaces by milling, to the machining of a worm and a wormwheel allows worm gears of an arbitrary profile to be obtained [6, 7]. In that case, however, worm and wormwheel convolution surfaces are defined as the envelopes of partial surfaces formed on each pass of the tool. This is a technology that enables the machining of gear elements using a universal tool, whose geometry is not associated with the surface being machined, on a universal CNC machine tool. The machining of a worm is relatively simple, as it involves a relative worm and tool movement, similarly as in the classical method. The difficulty lies in the calculation of the initial tool setup in each pass. Usually the geometry (surface equation), metrology (measurement of the outline of teeth) and the worm wheel technology are more difficult than in the case of worms. In the case of a wormwheel, the problem is not only to define the wormwheel tooth surface, but also to describe the motion of the tool relative to the wormwheel [8]. In this case the tool is a small-diameter ball-end finger mill, and it is also very important to select the correct number of tool passes at the machined profile height from the permissible profile lobing condition.

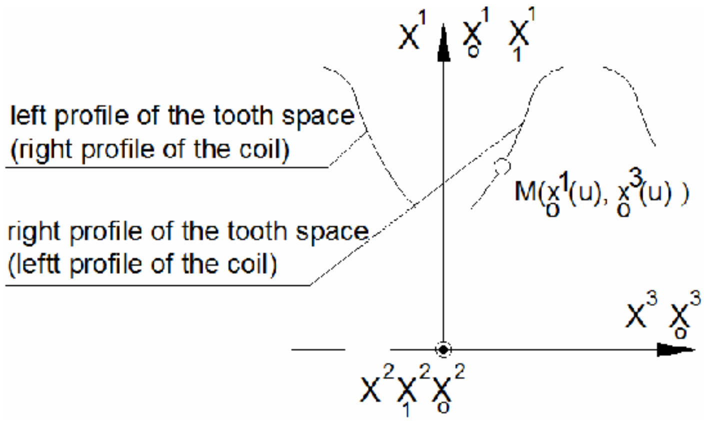

Fig. 1. Worm axial profile

The article presents the definition of the wormwheel tooth surface by an analytical-numerical method for the purposes of gear analysis and determination of the tool path for machining wormwheel teeth on a multi-axis CNC machine tool (e.g. 3-axis milling machine with rotary table).

Generating the worm surface

It is assumed that the wormwheel surface will be machined with a ball-end finger mill in multiple passes. In order to determine the path of the tool relative to the wormwheel tooth surface being machined, first this surface must be defined. It can be described as the envelope of the helical surface of the worm that the wormwheel is to mate with. The surface of a worm is generated by its axial profile with a helical motion (Fig. 1):

$$\bar{}_{1}(u, \quad v) = [3, \quad -v]\bar{}_{0} + [0, \quad 0, \quad \pm pv]^{T}, \tag{1}$$

where: u - worm axial profile parameter; v - parameter describing the helical worm profile motion; p - helical surface parameter; $\pm$ - the sign + or – for the right-hand and left-hand worm, respectively; - the subscripts identify the coordinate systems.

The wormwheel surface (Fig. 2) can be described by the system of equations (as the envelope of the family surfaces):

$$\bar{}_{2}(u, \quad v, \quad \xi) = [3, \quad i\xi]\left[1, \quad -\frac{\pi}{2}\right]\left([3, \quad \pi]\bar{}_{1} + [a, \quad 0, \quad -p\xi]^{T}\right), \tag{2}$$

$$f_2(u,\quad v,\quad \xi)=\frac{\partial \bar{.}_2}{\partial u}\,\frac{\partial \bar{.}_2}{\partial v}\,\frac{\partial \bar{.}_2}{\partial \xi}=0, \qquad (3)$$

where: i - gear ratio; ξ - parameter of relative worm and wormwheel motion.

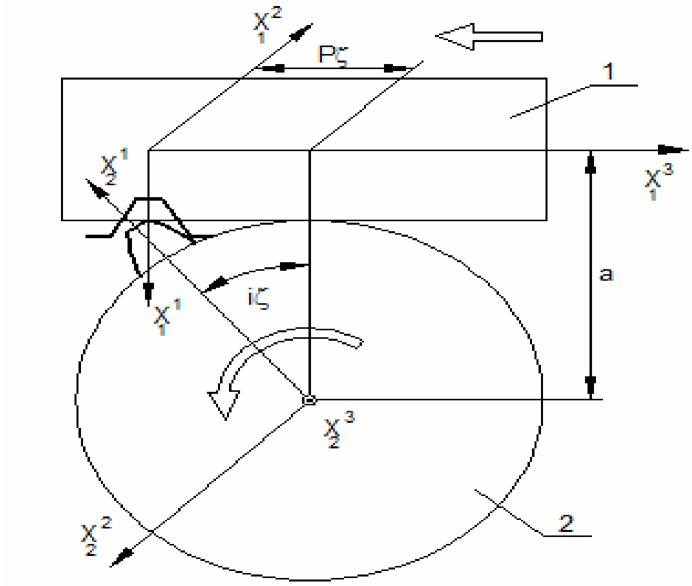

Fig. 2. Worm gear

Equation (3) is the envelope condition and can be transformed into a general form, as below:

$$\xi=\xi(u,\quad v). \qquad (4)$$

Equation (2), after substituting Equation (4) in it, can be generally written as a function of two parameters:

$$\bar{.}_2(u,\quad v,\quad \xi)=\bar{.}_2(u,\quad v,\quad \xi(u,\quad v))=\bar{.}_2(u,\quad v). \qquad (5)$$

This is the equation of wormwheel tooth flank surfaces. This equation enables one to define the surface as a set of points which can be used for the creation of the model of a tooth (and a wormwheel) in the form of a solid (for example, in the CATIA software program). The tooth model enables a machining program for a CNC machine tool to be automatically generated in a CAM program (such as EdgeCAM). The worm and wormwheel surface are in that case mutually enveloping. In such a case, it is not possible to modify the path of the tool centre, so it not possible to modify the wormwheel tooth flank surface. The wormwheel and its teeth are machined with a ball-end finger mill in multiple passes. The surface described by the motion of the tool sphere centre can be generally written with the equation

$$\bar{\ }_3 = \bar{\ }_2 + \rho \bar{\ }_2 , \qquad (6)$$

where: ρ - mill sphere radius.

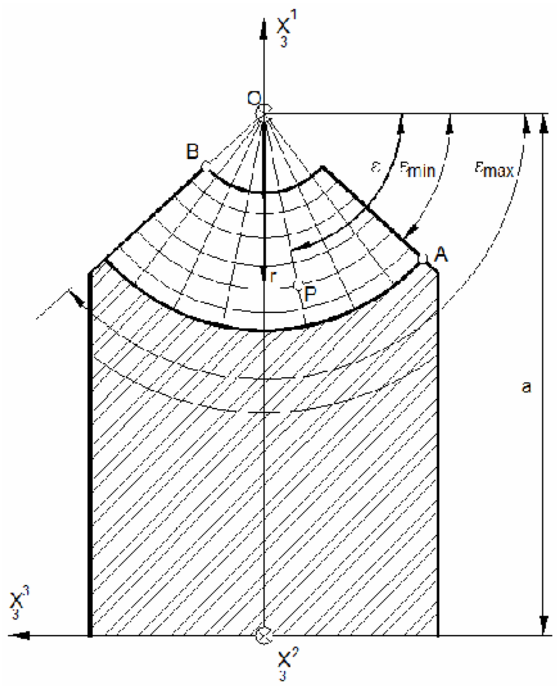

Fig. 3. Polar coordinates of the tool position centre in the wormwheel system

The normal to the wormwheel tooth surface

$$\bar{\ }_2 = \frac{\partial \bar{\ }_2}{\partial u} \times \frac{\partial \bar{\ }_2}{\partial v} , \qquad (7)$$

at the points of contact with the tool passes through the centre of the mill sphere.

It is assumed that the paths (curves) of the tool sphere centre in the wormwheel system will lie on cylindrical surfaces with an axis coinciding with the axis of the worm mating with the wormwheel (Fig. 3). The surface described by the motion of the tool sphere centre has been written in the form a set of points that project onto the grid of cylindrical coordinates, which is the axial plane of the wormwheel and is perpendicular to the worm axis. In order to pass on from the parametric coordinate grid, uv, to the coordinate grid, $r\varepsilon$, the method of approximating a surface with a set of planes has been used [4]. Through the nodal points of the grid $r\varepsilon$, ordinates are passed up to the intersection with the planes spread over the surface points (Fig. 4) determined for the points of the grid uv.

The surface points lying on the lines with a fixed radius (Fig. 5, Fig. 6):

$$r = \text{const} , \qquad (8)$$

enable the programming of the tool movement in the wormwheel machining process. The modification of these lines enables the longitudinal modification of wormwheel teeth.

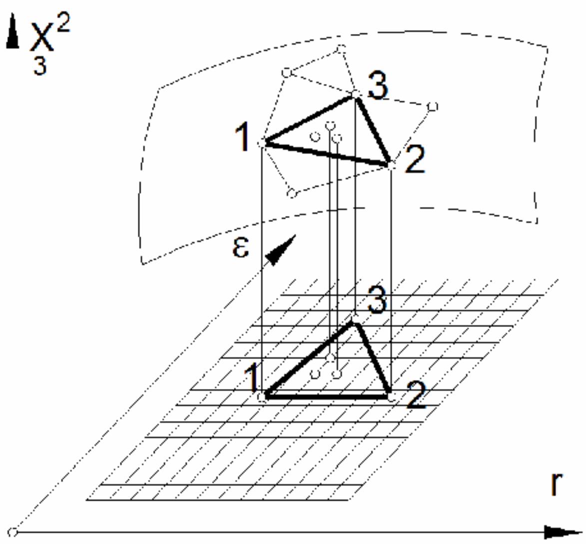

Fig. 4. Approximation of a surface with a set of planes

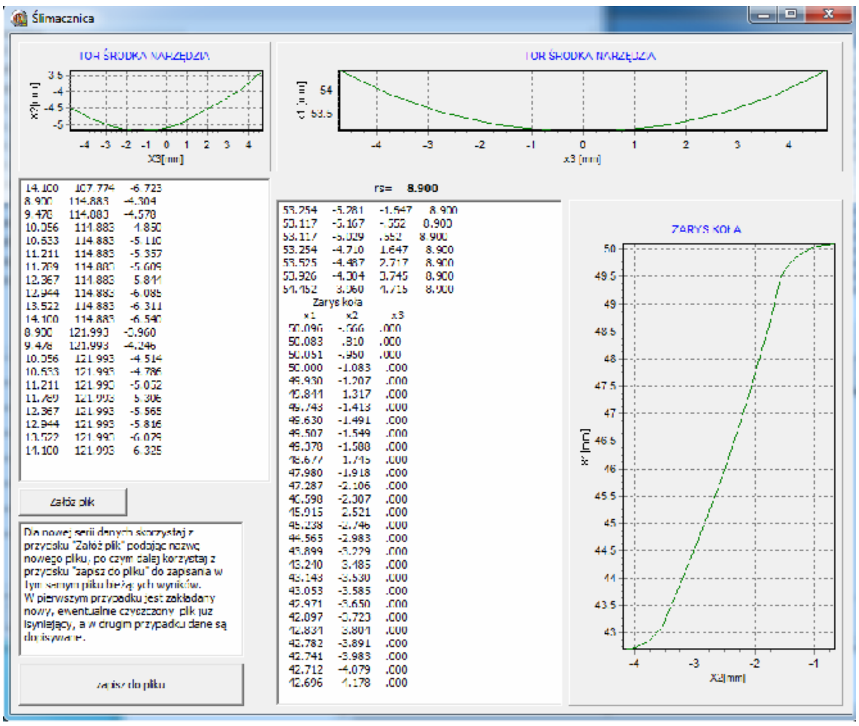

Fig. 5. Curve representing the path of the finger mill sphere centre in the machine wormwheel's system (a right-hand worm) at the tooth top

The modification of the wormwheel tooth surface at the profile height can be achieved through the modification of the worm axial profile or through the modification of the lines

$$\varepsilon = \text{const} . \tag{9}$$

It means that the worm, which was the basis for determining the wormwheel tooth surface, can be different from the worm that this wormwheel will form the gear with. This difference results also from the need for allowing for clearances in the gear worm between the worm and the wormwheel.

The longitudinal modification of the wormwheel tooth surface allows the trace of worm and wormwheel tooth abutment to be located. In that case, the analysis of the wheels of the worm gear needs to be made [9,10]. Transmissions with the abutment trace location are insensitive to assembly errors. The analysis also allows the determination of transmission errors.

Currently, modern CNC machine tools offer the capability to machine geometrically complex surfaces using incorporated functions for the interpolation of any curves [11]. For example, the spline interpolation automatically creates a curve that smoothly traces specified points, and thus enables a high-speed and high-accuracy machining for free shapes along a smoothly curved tool path. The fine spline function works with spline interpolation and automatically corrects the shape of a spline curve, as required, to make the path of the curve smoother. The NURBS interpolation function provides interpolation by performing NURBS-defined CNC internal computations on the command issued from the CAD/CAM system in the NURBS format. With this optional function, a very smooth interpolation path can be obtained since the interpolation process is performed directly without dividing a NURBS-formatted free-form curve into minute line segments.

Conclusion

The application of the method of worm and wormwheel machining with a ball-end finger-type mill in multiple passes enables the formation of worm gears of any arbitrary profile. However, due to the need for multiple accurate setting up of the tool, this method can only be accomplished on CNC machine tools. The profile of the tool is independent of the profile being machined, and these types of cutters are general-purpose commercial tools. Small-diameter finger cutters are manufactured also as fully carbide tools, and thus they can be used for machining hard-machinable materials (such as titanium, or nickel alloys) which are increasingly widely applied in the aircraft and the power industries, or for machining toughened materials (eliminating grinding). The advances in the technology of machining worm gears of an arbitrary profile will contribute to the development of new designs of concave-convex worm gears. Especially in the case of machining wormwheels on a universal CNC machine tool, instead of machining on a special gear hobbing machine, the cost-effectiveness of this machining may be high. In the case of machining a wormwheel with a hob there is a continuous division, whereas in the case in question an intermittent division occurs. In view of the large number (around several dozen) of wormwheel teeth, the machining of a wormwheel with a finger cutter could be less efficient than the machining with a hob (in the case of a worm with merely a few threads this issue will not be so important at all). However, in the case of small-series production, large-sized gears, or the need for making an additional gear, this method will be the best.

The surface and axial profile of worm coils are defined by the analytical method. Defining the worm axial profile is important owing to the ease of measuring this profile and the ease of setting the mill in the worm axial profile. For defining the wormwheel surface, an algorithm for the approximation of a surface with a set of planes was used. The accuracy of approximation depends on the density of the coordinate grid, whose nodal points project onto the surface points. A circular coordinate grid was taken for defining the tool paths during machining of wormwheel tooth surfaces lying on concentric cylindrical surfaces. The cutter diameter and the number of passes for convolution profile machining are selected based on the assumed profile angularity condition and the condition of machining the profile over its whole height.

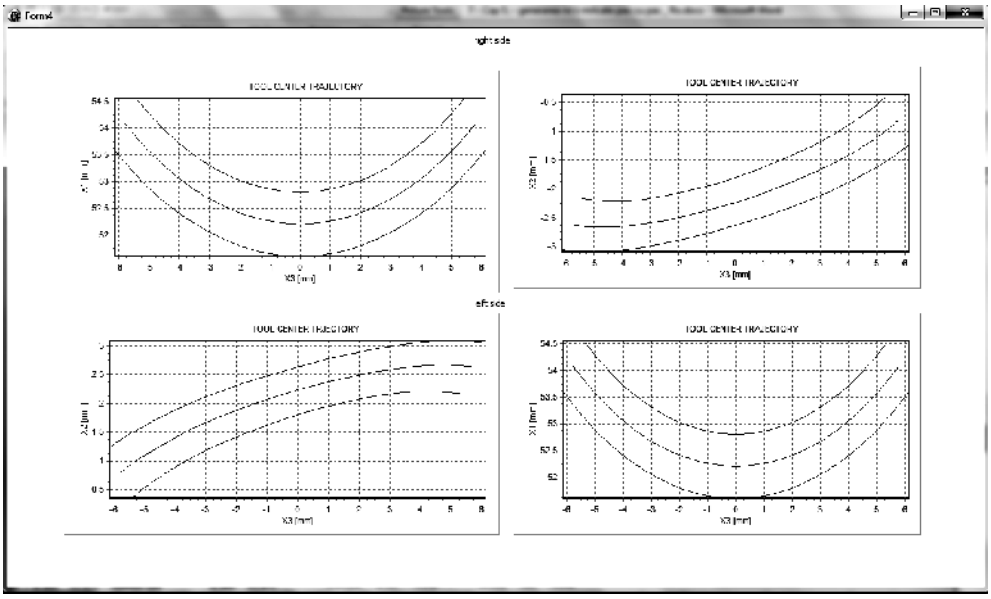

Fig. 6. An example of curves representing the path of the finger mill sphere centre in the machine wormwheel's system (a right-hand worm) for left and right side of the tooth

Using the technology of machining worm and wormwheel surfaces with a ball-end finger-type mill in multiple passes enables the longitudinal and transverse modification of the teeth, which was not possible with the use of hobs. Tooth modification should be preceded by the synthesis and analysis of the worm gear meshing. As a result, there is a possibility of cutting approximate gears, theoretically with a point tooth contact, which has been so far the feature characteristic of bevel gears.

This technology can also be applied to treatment of other types of gears.

References:

1. Litvin F.L. Gear Geometry and Applied Theory / Litvin, F.L. & Fluentes A. – New York: Cambridge University Press, 2004. – 800 p.

2. Dudas I. Csigahajtasok elmelete es gyartasa / Dudas I. – Budapest: Muszaki Kiado, 2007. – 335 p.

3. Marciniak T. Przekładnie ślimakowe walcowe / T. Marciniak. - Warszawa: WNT, 2001. – 194 p.

4. Nieszporek T. Podstawy konstrukcji narzędzi skrawających do obróbki walcowych uzębień zewnętrznych / Nieszporek T. – Częstochowa: Politechnika Częstochowska, 2004. – 243 p.

5. Research Regarding Analitycal and Numerical Determination of Worm Gear with Parabolic Profil / Boca V., Gyenge Cs., Gurzau O.M. et al. // Cluj Napoca: Proceedings of the International Conference Modern Technologies in Manufacturing, Gyenge, Cs. (Ed.), October 2009, Technical University of Cluj-Napoca. – 2009. – 13-20 p.

6. Nieszporek T. Generation of the worm helical surface / Nieszporek T., Łyczko K. // Bucuresti: Proceedings In Manufacturing Systems. – 2010. – Vol. 5. – No. 1. – 31-34 p.

7. Nieszporek T. A New metod of manufacturing the worm gear with concave profile / Nieszporek T., Boca V. // Cluj Napoca: Proceedings of the International Conference Modern Technologies in Manufacturing, Gyenge, Cs. (Ed.), October 2011, Technical University of Cluj-Napoca. – 2011. – 218-221 p.

8. Wojciechowski J. Metody programowania obróbki powierzchni śrubowych o zmiennym skoku na obrabiarkach wieloosiowych / Wojciechowski J. – Warszawa: Oficyna Wydawnicza Politechniki Warszawskiej. – 2001. – 143 p.

9. Litvin F.L. Development of Gear Technology and Theory of Gearing / Litvin F.L. – Washington and Maryland: National Aeronautics and Space Administration, Lewis Research Center, 1997. – 113 p.

10. Nieszporek T. The Analysis of the Meshing of Worm and Worm Gear Surfaces / T. Nieszporek, P. Boral // Cluj Napoca: Proceedings of the International Conference Modern Technologies in Manufacturing, Gyenge, Cs. (Ed.), October 2007, Technical University of Cluj-Napoca. – 2007. – 315-318 p.

11. Yamazaki Mazak Corporation. Programming Manual for Mazatrol Fusion 640M Mazatrol Fusion 640 5X Mazatrol Fusion 640M NEXUS Programming EIA/ISO. – Manual H735PB0031E, Yamazaki Mazak Corporation, Japan, 2004.

Надійшла до редколегії 21.01.2012.

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ЧИСЛЕННЫЙ МЕТОД ОПРЕДЕЛЕНИЯ ПОВЕРХНОСТИ ЧЕРВЯЧНОГО КОЛЕСА

Для проверки точности изготовления червяка или червячного колеса и анализа зацепления, нужно определить эти поверхности. Производящая поверхность режущего инструмента для обработки червячного колеса должна быть согласной с поверхностью червяка, который восстанавливает с этим колесом червячную передачу. Это условие не всегда исполнено, и поверхность зубьев червячного колеса очень часто описывается приблизительно. В этой статье предлагается численный метод определения червячного колеса нарезанного шаровидной пальцевой фрезой небольшого диаметра за счет множества проходов по высоте профиля зубьев. По этому методу можно нарезать червячную передачу произвольного профиля. Достоинством является возможность разработки новых видов червячных передач, особенно с круговым профилем зубьев. В случае необходимости нарезания крупномодульных передач или мелкосерийного производства этот метод будет очень полезным. Эта технология требует станков с ЧПУ.

Ключевые слова: *червячная передача, червячное колесо, шаровидная пальцевая фреза.*

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ЧИСЕЛЬНИЙ МЕТОД ВИЗНАЧЕННЯ ПОВЕРХНІ ЧЕРВ'ЯЧНОГО КОЛЕСА

Для перевірки точності виготовлення черв'яка або черв'ячного колеса та аналізу зачеплення, необхідно визначити ці поверхні. Твірна поверхня ріжучого інструменту для обробки черв'ячного колеса повинна бути узгоджена з поверхнею черв'яка, який відновлює з цим колесом черв'ячну передачу. Ця умова не завжди виконана і поверхня зубів черв'ячного колеса дуже часто описується приблизно. У цій статті пропонується чисельний метод визначення черв'ячного колеса нарізаного кулястою пальцьовою фрезою невеликого діаметру за рахунок багаторазового проходу по висоті профілю зубів. Ця технологія дозволяє нарізати черв'ячну передачу довільного профілю. Це дає можливість розробки нових видів черв'ячних передач, особливо з круговим профілем зубів. Цей метод буде дуже корисним у випадку необхідності нарізування крупномодульних передач або дрібносерійного виробництва. Ця технологія вимагає використання верстатів з ЧПУ.

Ключові слова*: черв'ячна передача, черв'ячне колесо, куляста пальцева фреза.*